

# An improved method for estimating source densities using the temporal distribution of Cosmological Transients

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## ABSTRACT

It has been shown that the observed temporal distribution of transient events in the cosmos can be used to constrain their rate density. Here we show that the peak flux–observation time relation takes the form of a power law that is invariant to the luminosity distribution of the sources, and that the method can be greatly improved by invoking time reversal invariance and the temporal cosmological principle. We demonstrate how the method can be used to constrain distributions of transient events, by applying it to *Swift* gamma-ray burst data and show that the peak flux–observation time relation is in good agreement with recent estimates of source parameters. We additionally show that the intrinsic time dependence allows the method to be used as a predictive tool. Within the next year of *Swift* observation, we find a 50% chance of obtaining a peak flux

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greater than that of GRB 060017 – the highest *Swift* peak flux to date – and the same probability of detecting a burst with peak flux  $> 100 \text{ photons s}^{-1}\text{cm}^{-2}$  within 6 years.

*Subject headings:* gamma-rays: bursts –gravitational waves – cosmology: miscellaneous

## 1. INTRODUCTION

The brightness distribution of cosmological sources is conventionally used to constrain the luminosity function of the sources, their evolution in density (Peebles 1993) and, for transient sources, their rate density (Schmidt 2001; Sethi & Bhargavi 2001; Totani 1997). This method is applicable both to long-lived sources such as galaxies and to transient events such as supernovae and gamma-ray bursts (GRBs). Estimates are obtained by fitting the number – brightness distribution to models that include luminosity, source density and evolution effects. In the case of transient events an additional parameter is available – the event arrival times.

The temporal distribution of transient astrophysical populations of events has been described by the ‘probability event horizon’ (PEH) concept of Coward & Burman (2005). This method establishes a temporal dependence by noting the occurrences of successively brighter events in a time series. By utilizing the fact that the rarest events will preferentially occur after the longest observational periods, it produces a data set with a unique statistical

signature.

Here we show that a well-defined observation-time dependence is an intrinsic feature of the source distribution of events. Using Swift GRB data we demonstrate how this property can be used to constrain source distributions. We start by presenting an analytical derivation of the peak flux–observation time relation,  $P(T)$ , for sources which are uniformly distributed in Euclidean space and then describe its extension to cosmological models (§2). We derive a simple power-law relation for  $P(T)$  that is invariant to the luminosity distribution of events.

We then utilize the PEH technique to show how  $P(T)$  data can be extracted from a distribution of peak fluxes (§3). We show that the PEH method can be greatly improved by invoking time reversal invariance and the temporal cosmological principle: for time scales that are short compared to the age of the Universe, a distribution of independent events is invariant with respect to temporal direction and there is nothing special about the time when we switch on our detector.

As a test, we apply the  $P(T)$  relation to *Swift* long-GRB data. We demonstrate that the technique can be used as a probe of the event rate density and luminosity distribution of the sources (§4), and as a tool to predict the likelihood of future high peak flux events (§5).

## 2. THE PEAK FLUX – OBSERVATION TIME RELATION

In this paper we will define an *event* to be an astrophysical transient occurrence with a duration much less than the period of observation. Examples are GRBs and gravitational

wave burst sources such as coalescing compact binaries or core-collapse supernovae.

Consider a distribution of events defined within a Euclidean space by an event rate density  $r_0$  and a luminosity function<sup>1</sup>  $\phi(L)$  ( $L_{\min} \leq L \leq L_{\max}$ ). The observed peak flux, or ‘brightness’, distribution of events over an observation time  $T$  is a convolution of the radial distribution of the sources and their luminosity function. For peak fluxes (photons  $\text{cm}^{-2}\text{s}^{-1}$ ) between  $P$  and  $P + dP$ :

$$dN(P) = 4\pi T \int_{L_{\min}}^{L_{\max}} \phi(L) dL r_0 r^2 dr, \quad (1)$$

with  $r = (L/4\pi P)^{1/2}$ . The total number of events observed in time  $T$  with a peak flux greater than  $P$  is given by :

$$N(> P) = T \Delta\Omega \int_{L_{\min}}^{L_{\max}} \phi(L) dL \int_0^{\sqrt{L/4\pi P}} r_0 r^2 dr, \quad (2)$$

where the average solid angle covered on the sky has been accounted for by  $\Delta\Omega/4\pi$ . The upper limit in the integration over  $r$  is the maximum distance for which an event with luminosity  $L$  produces a peak flux  $P$ .

For  $r_0$  and  $\phi(L)$  independent of position, integrating over the radial distance yields:

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<sup>1</sup>We use here the luminosity function for GRB sources,  $\phi(L)$ , which includes a normalization constant to ensure that it integrates to unity over the range of source luminosities. This means that  $\phi(L)$  has units of inverse luminosity—see for example, Porciani & Madau (2001).

$$N(> P) = \frac{Tr_0\Delta\Omega/4\pi}{3\sqrt{4\pi}} P^{-3/2} \int_{L_{\min}}^{L_{\max}} \phi(L) L^{3/2} dL. \quad (3)$$

This is the familiar log  $N$ –log  $P$  relation,  $N(> P) \propto P^{-3/2}$ , a power law independent of the form of the luminosity function (Horack, Emslie & Meegan 1994).

To introduce the temporal distribution of events, we note that, as the events are independent of each other, the individual events will follow a Poisson distribution in time. Therefore, the temporal separation between events will follow an exponential distribution, defined by a mean event rate  $R(r) = r_0(4/3)\pi r^3$  for events out to  $r$ . The probability for at least one event  $> P$  to occur in a volume bounded by  $r$  during an observation time  $T$  at constant probability  $\epsilon$  is given by:

$$\mathcal{P}(n \geq 1; R(r), T) = 1 - e^{R(r)T} = \epsilon. \quad (4)$$

For this equation to remain satisfied with increasing observation time:

$$N(> P) = R(r)T = |\ln(1 - \epsilon)|. \quad (5)$$

Equations (3) and (5) for  $N(> P)$  combine to give the relation for the evolution of brightness as a function of observation time:

$$P(T) = \left( \frac{r_0\Delta\Omega/4\pi}{3\sqrt{4\pi}|\ln(1 - \epsilon)|} \right)^{2/3} \left[ \int_{L_{\min}}^{L_{\max}} \phi(L) L^{3/2} dL \right]^{2/3} T^{2/3}. \quad (6)$$

This relation shows that for a simple Euclidean geometry, a  $\log P$ – $\log T$  distribution will have a slope of  $2/3$ , independent of the form of the luminosity function. One can consider that changes in  $r_0$  create a horizontal offset in the  $\log P$ – $\log T$  distribution, while changes in the integrated luminosity create a vertical offset. However, the slope is fixed by the 3-D Euclidean geometry.

We can use the  $\log P$ – $\log T$  relation to produce curves defining the probability,  $\epsilon$ , of obtaining some value of peak flux,  $P$ , within an observation-time,  $T$ , for a given  $r_0$  and  $\phi(L)$ .

For a cosmological distribution of sources, equation (6) must be modified to allow for cosmic evolution. A standard Friedman cosmology can be used to define a differential event rate,  $dR(z)$ , in the redshift shell  $z$  to  $z+dz$ . The luminosity and flux will be related through  $z$  by a luminosity distance  $d_L^2(z)$  (see for example Coward & Burman (2005) or Porciani & Madau (2001)). In this case, solving equation (5) numerically, with  $P = L/4\pi d_L^2(z)$ , will yield the cosmological  $\log P$ – $\log T$  relation.

### 3. AN ENHANCED PEH FILTER

To utilize the time domain, we use the probability event horizon (PEH) filter of Coward & Burman (2005) to produce  $P(T)$  time data. The PEH filter is a tool that exploits the temporal information encoded in a time series of transient events and works by recording successively brighter events in a time series. Howell et al. (2007) demonstrated that the unique statistical signature of events filtered in this way could be exploited to obtain rate estimates of

transient events. However, the significant probability of a bright event occurring early in an observational period meant that only a small fraction of data was used by the method. As a result, large uncertainties were obtained in the estimates. There are however, two ways in which the amount of usable data can be increased.

Firstly, the temporal cosmological principle implies that the PEH signature of a transient population of events is independent of when a detector is switched on. Secondly, time reversal invariance allows the PEH filter to be applied to a data set in both temporal directions. Thus, a time series of events can be treated as a closed loop which can be interrogated in both directions. The observational period is now defined as the total length of the loop. The start time for the PEH analysis is now arbitrary so, without loss of generality, we can choose any start time. This allows the PEH filter to be applied in such a way that the brightest event can be set as the final event in a series. This ensures that the PEH filter is applied to the full data stream and the process can be repeated in each direction, increasing the quantity of PEH data. We refer to these techniques as ‘from max’ plus ‘time reversal’ (FMTR). We show below how FMTR increases the PEH sample and significantly improves the statistical resolution when applied to the *Swift* data.

#### 4. APPLICATION TO *SWIFT* DATA

In this section, we will apply the  $\log P$ – $\log T$  relation to a cosmological population of long GRBs. To account for the event rate and luminosity function, we will use estimates

based on recent studies. Using the FMTR method, we will extract a time-dependent sample of GRB peak fluxes from the *Swift* data, and demonstrate how it can be constrained by a  $\log P$ – $\log T$  fit.

For our long-GRB peak flux sample we use data recorded by the *Swift*<sup>2</sup> satellite between 2004 December and 2007 April. We consider only bursts with confirmed peak fluxes detected within the 15–150 keV band of the Burst Alert Telescope (BAT) and with  $T_{90} \geq 2\text{ s}$  (a  $T_{90}$  duration is the interval in which a signal contains 90% of its total observed counts). The total sample consists of 190 peak fluxes.

Figure 1A displays the *Swift* peak flux distribution of long GRBs as a time series. It is apparent that as observation time increases, there is an greater probability of a bright event. By extracting successively brighter events as a function of observation time, the PEH filter samples events from the low probability tail of the distribution. The strong brightness–time dependence of these events creates a unique statistical signature which can be modeled by the  $\log P - \log T$  relation.

In Table 1 we show the PEH filtered data. It is apparent how the time intervals between successive events increase with observation time. This is a result of a progressive sampling of the rarer events of the distribution.

To apply the  $\log P$ – $\log T$  relation to the filtered data we must first set up a model to account for the source rate evolution and luminosity distribution. We use a model from the recent study of Guetta & Piran (2007). They employ a ‘flat- $\Lambda$ ’ cosmological model and

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<sup>2</sup>This data can be obtained from the *Swift* website [http://swift.gsfc.nasa.gov/docs/swift/archive/grb\\_table.html](http://swift.gsfc.nasa.gov/docs/swift/archive/grb_table.html)



$H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$  for the Hubble parameter at the present epoch. For the isotropic luminosity function of GRB peak luminosities,  $\phi(L)$ , they use a broken power law form based on the work of Schmidt (2001). Assuming that the rate of GRBs traces the global star formation history of the Universe, they employ a number of different star formation rate models. For each, they determine best fitted values for the luminosity function and event rate density. For this study, we use their model (i) parameters, which are based on the SF2 star formation rate model of Porciani & Madau (2001). These parameters include fitted values for the luminosity function and a local event rate density  $r_0 = 0.1 \text{ Gpc}^{-3} \text{ yr}^{-1}$ . To account for the average solid angle covered on the sky by *Swift*, we use a value of  $\Delta\Omega = 1.33$  (Band 2005).

Figure 1B shows a  $\log P$ – $\log T$  fit to the PEH filtered data (shown by squares), using the fitting parameters of Guetta & Piran (2007): i.e. there are no free parameters in the comparison between theory and observation. We define a 90% confidence band – shown by the shaded area – corresponding to the  $\epsilon = 95\%$  (top) and  $\epsilon = 5\%$  (bottom) probabilities of detecting at least one event within an observation time  $T$ . We see that the data is well constrained. The fit shows that by using only a small sample of the brightest events, it is possible to extract the geometrical signature of the source population and to test estimates of the luminosity function and rate density of events.

The dashed lines of Figure 1B show the 90% confidence band corresponding to the Euclidean model using equation (6). We see that two of the first few events lie outside the Euclidean curves but are constrained by the cosmological model. These events, occurring

at early observation times, most likely result from sources at large cosmological distances. The very bright events at late observation times are more probable – it is apparent that the Euclidean and cosmological curves begin to converge in this regime. To account for non-uniformly distributed sources, the method could be refined to take into account the spatial distribution of potential host galaxies.

To test the power-law dependence in equation (6), we have performed least-squares fitting to the PEH filtered data using the Euclidean  $\log P$ – $\log T$  curve as a linear regression model. By setting the power as a single free parameter, we obtained a value of  $0.67 \pm 0.02$ , confirming the  $2/3$  slope derived in section 2.

## 5. A PREDICTIVE APPLICATION OF THE LOG P – LOG T RELATION

Figure 2 illustrates how the  $\log P$ – $\log T$  relation can be used as a predictive tool. By mapping the temporal evolution of detection probability, the maximum brightness of future events can be constrained (Coward et al. 2005). As in Fig. 1B, the shaded areas show the  $\log P$ – $\log T$  detection confidence bands corresponding to different probability values (shown in the legend). The current *Swift* observation time (398 days) is shown by the vertical solid line. For predictive applications it is essential that the true temporal sequence of events is retained. Therefore, rather than using the FMTR technique, we apply the unmodified PEH filter from the time of the first event. Comparing with Fig. 1B, we see that the FMTR method has increased the sample by 220%, of which 80% is gained by incorporating time reversal. Using 100 Monte-Carlo simulations, we find a mean fractional increase in data of

$\sim 35\%$ , compared to the unmodified PEH filter. The larger than expected filter output using *Swift* data, implies that the FMTR method can be further optimally tuned.

The plot shows that after 3 days of operation, there was a  $< 5\%$  probability of detecting an event with peak flux equal to the first in the PEH sample, GRB 041123. The Poisson probability of detecting at least one event within a Gpc at this time is  $\sim 2 \times 10^{-6}$ . This implies that this event occurred at a considerable cosmological distance. The event GRB 050525A is the brightest long GRB with a secure redshift,  $z = 0.606$  – the Poisson probability of at least one event within this volume is 35%. The next brightest event, GRB 060017, is the most intense burst, in terms of peak flux, detected by Swift.

As a demonstration of the predictive nature of the the  $\log P - \log T$  relation, Figure 2 shows that there is a 50% probability of obtaining an event with a peak flux greater than that of GRB 060017 within the next year and an 80% probability within 5 years.

The curve predicts that there is a 50% (80%) probability of obtaining a burst with peak flux  $> 60 \text{ photons s}^{-1}\text{cm}^{-2}$  within 2 (8) years. To determine the feasibility of this prediction, we consider again GRB 050525A, which had a peak flux of  $42.3 \text{ photons s}^{-1}\text{cm}^{-2}$ . Using this burst’s redshift and converting to a peak luminosity, we find that an equivalent burst would have to occur within  $z \approx 0.5$  to produce a peak flux of this level. The Poisson probability of at least one event at this distance within the next two years is  $\sim 90\%$  (99%). If we consider a burst with a peak flux of  $100 \text{ photons s}^{-1}\text{cm}^{-2}$ , we find that an event of this peak flux is 50% probable within 6 years. Such an event would correspond to a GRB 050525A-equivalent

burst occurring within  $z \approx 0.4$ , for which the probability is 99%.

The  $\log P - \log T$  technique naturally uses the brightest events of a data set. As the probability of obtaining a GRB afterglow increases with peak flux, the method can be used to predict the expected occurrence of events at low  $z$ .

## 6. SUMMARY

We have provided a clear demonstration of the  $\log P - \log T$  relation by applying the PEH filter to the *Swift* GRB peak flux distribution. A  $\log P - \log T$  model with no free parameters was fitted to filtered data confirming the power law  $P \propto T^{2/3}$  in the Euclidean limit; the power law is independent of the form of the luminosity distribution.

The FMTR method significantly improves the PEH method, which was previously disadvantaged by using only a small fraction of a data stream. We have shown that FMTR enables the PEH filter to use over 8% of the available data, making it a practical tool for cosmology.

We have shown that the PEH technique can be used as a predictive tool. Comparing observation with prediction provides an additional means to test rate estimates and evaluate source parameters such as the limits of the luminosity distribution.

In a future study, we intend to apply the FMTR method to both the *Swift* and *BATSE* GRB data. We will investigate the efficiency of the method in determining constraints on

the rate density and limits of the luminosity function.

## ACKNOWLEDGMENTS

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GRB	Peak Flux	Redshift	Observation Time
	( $\text{photonss}^{-1}\text{cm}^{-2}$ )		(days)
060202	0.5		8
060203	0.6		9
060204B	1.3		10
060206	2.8	4.045	12
060223B	2.9		29
060306	6.1		42
060418	6.7	1.49	84
060510A	17.0		106
061121	21.1	1.314	297
050219B	25.4		506
050525A	42.3	0.606	602
060117	48.9		839
060111A	1.72		4
060110	1.9		5
060105	7.5		10
060603	27.6	2.821	227

Table 1: Data extracted from the *Swift* peak flux distribution of long GRBs using the PEH filter. The observation time is determined by treating the time series as a closed loop and setting the last event in the series to be the brightest burst. The lower set of data was obtained by invoking time reversal. A  $\log P - \log T$  fit to this data is shown in Fig. 1

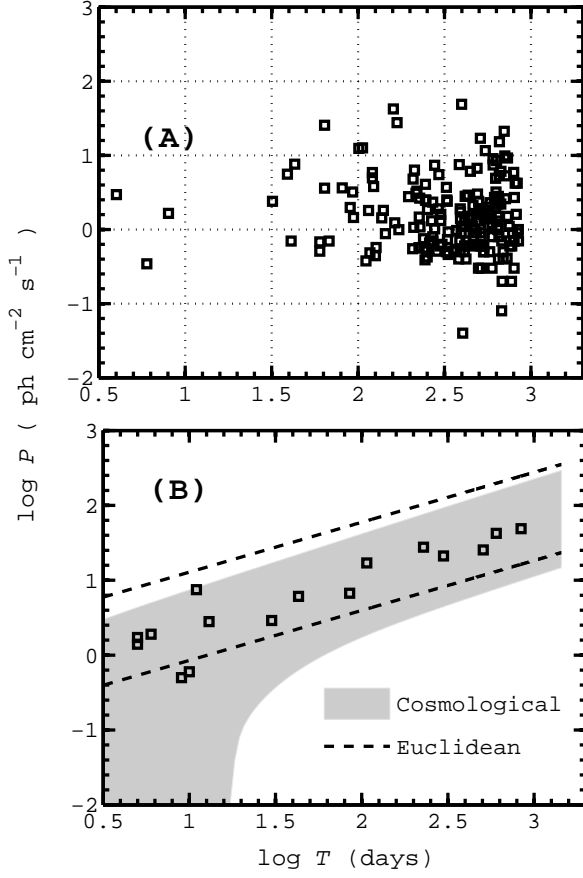


Fig. 1.— Panel (A) shows the *Swift* peak flux distribution as a time series. It is evident that as observation time increases the probability of a bright event increases. Panel (B) uses a PEH filter to extract the geometrical signature of the GRB distribution (shown as squares). Assuming an event rate of  $0.1 \text{ Gpc}^{-3} \text{ yr}^{-1}$  (Guetta & Piran 2007), the shaded area shows the cosmology dependent  $\log P$ – $\log T$  model corresponding to a (5 – 95)% confidence band. The equivalent model for a Euclidean geometry is shown by the dashed curves – the two outliers, which have no associated redshifts, probably result from distant cosmological events.



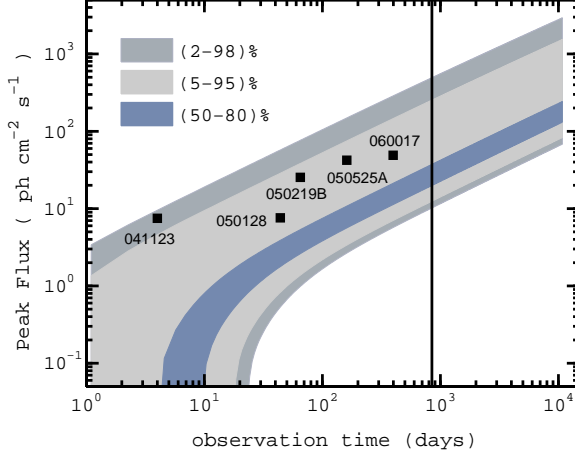


Fig. 2.— The  $\log P$ – $\log T$  relation used as a predictive tool. The successive maximum peak fluxes detected by Swift since the start of operation are shown by squares. The shaded areas show detection confidence bands corresponding to different probability values (shown in the legend). The current *Swift* observation time (398 days) is shown by the solid line. The plot shows that after 3 days of operation, there was a  $< 5\%$  probability of detecting an event with peak flux equal to the first in the PEH sample, GRB 041123. We see that within the next year, there is a 50% chance of obtaining a peak flux greater than for GRB 060017 – the most intense burst (in peak flux) detected by Swift – and the same probability of obtaining a burst with peak flux  $> 100 \text{ photons s}^{-1}\text{cm}^{-2}$  within 6 years.